**Refining RFT 5.9: Addressing Void Lensing, Clusters, and Gravitational Waves**

**Void Lensing Fix**

Cosmic voids are vast underdense regions that produce only weak gravitational lensing signals. Large voids have mean density contrasts on the order of δ ~ –0.5 (50% of the cosmic mean)​

[inis.iaea.org](https://inis.iaea.org/records/b75pp-pgr74/files/55063467.pdf?download=1#:~:text=,Wandelt%2C%20Universal%20Density%20Profile)

, yielding slight negative lensing convergence (κ) values in line of sight projections. RFT 5.9’s prior parameters were overestimating this void lensing effect, so we will calibrate the model’s function *f*(E, ρ, v) to better match observations. Key steps include:

* **Simulation & Parameter Tuning** – Run a Gadget-4 N-body simulation (100 Mpc/h box, 512^3 particles, z=0) with adjusted RFT parameters. We decrease the dimensionless coupling *k* to ~0.5–0.6 and increase the critical density threshold ρ<sub>crit</sub> to ~9–10×10^−27 kg/m³ (about 5–6 times the mean cosmic density) in *f*(E, ρ, v). These tweaks are designed to suppress the excessive lensing signal within voids by making RFT’s modifications “turn off” in extremely low-density environments.
* **Void Profile Analysis** – Identify voids in the simulation and measure their density profiles and lensing convergence. We expect the average void density contrast to level out around δ ≈ –0.4 to –0.5 at void centers, and the minimum convergence in stacked void lensing maps to be κ<sub>min</sub> ≈ –4×10^−4 to –5×10^−4 (i.e. a few ×10^−4 negative) after the parameter adjustment. These values are consistent with cosmic void lensing observations, which find only very slight “de-lensing” through void cores​

[academic.oup.com](https://academic.oup.com/mnras/article/534/3/2328/7774405#:~:text=For%20most%20voids%2C%20the%20low,sight%20through%20the%20void%20centre)

. By contrast, earlier RFT 5.9 predictions overshot these signals, so hitting this target range will indicate a better fit.

* **Comparison with Data** – Cross-check the simulation results against real void catalog data and weak lensing measurements. We will use voids identified in SDSS DR12 and DESI galaxy surveys as a baseline for void sizes and density contrasts, and compare the lensing profiles to measurements from DES and KiDS surveys. Observationally, the void lensing amplitude is detected at modest significance and is generally in agreement with ΛCDM predictions within ~2σ​

[arxiv.org](https://arxiv.org/abs/2203.11306#:~:text=expected%20%24A_,weaker%20than%20expected%20from%20MICE)

. Our goal is to ensure the refined RFT 5.9 predictions fall within the 1–2σ observational uncertainty band of these measurements, eliminating the previous overprediction. For instance, DES Year 3 analyses of voids and superclusters find a combined lensing signal amplitude A<sub>κ</sub> ≈ 0.82±0.08, slightly (2.3σ) lower than expected but broadly consistent​

[arxiv.org](https://arxiv.org/abs/2203.11306#:~:text=expected%20%24A_,weaker%20than%20expected%20from%20MICE)

. By matching such data, the adjusted RFT 5.9 will remain empirically viable on void scales.

**Cluster Dynamics Improvement**

Massive galaxy clusters, especially energetic merging systems like “El Gordo” and the “Sausage” cluster (CIZA J2242.8+5301), pose another test for RFT 5.9. These clusters have extremely deep gravitational potentials and high galaxy velocity dispersions (on the order of 1000–1300 km/s) that were challenging to reproduce without invoking a large neutrino mass in previous models. Here we enhance the RFT parameters to capture cluster dynamics *without* relying on heavy neutrinos, by making gravity stronger in cluster cores. The approach:

* **Hydrodynamic Cluster Simulations** – Using Gadget-4 with hydrodynamics, we simulate the two noted clusters. We increase the critical energy density threshold E<sub>crit</sub> (in *f*(E, ρ, v)) to 5×10^−30 (in the chosen units) to deepen the gravitational wells in high-energy regions like cluster centers. This effectively boosts the gravitational field inside cluster cores under RFT. The expected outcome is higher galaxy velocities and a tighter potential well, mimicking the dynamical mass of these clusters. For example, El Gordo (ACT-CL J0102–4915) has an observed galaxy velocity dispersion of σ<sub>gal</sub> ≈ 1321±106 km/s​

[ntrs.nasa.gov](https://ntrs.nasa.gov/citations/20120002647#:~:text=significant%20Sunyaev,X%29)

, and the Sausage cluster’s two subclusters have dispersions ~1160 km/s and ~1080 km/s​

[arxiv.org](https://arxiv.org/abs/1410.2893#:~:text=north%20and%20south%20subclusters%20have,These%20correspond%20to%20masses%20of)

(roughly ~1000–1100 km/s). The simulations with elevated E<sub>crit</sub> should approach these values organically, indicating that RFT’s altered gravity can account for the cluster dynamics.

* **Mass Distribution and Lensing** – We generate projected mass (convergence κ) maps from the simulations to compare with strong and weak lensing observations (e.g. from HST and Chandra X-ray lensing analyses). The aim is for the peak convergence in the cluster cores to be κ ~0.8–1.0 within a ~100 kpc radius, matching the surface mass density inferred from observations. Such high central κ values are typical for very massive clusters’ lensing reconstructions​

[arxiv.org](https://arxiv.org/html/2403.06245v1#:~:text=Image%3A%20Refer%20to%20caption%20Figure,center%20of%20mass%20and%20light)

, indicating near-critical surface density in the core. Achieving κ peaks of order unity in RFT 5.9 would confirm the model can produce the necessary concentration of mass. We will verify that the simulated κ-map for El Gordo, for instance, shows a central value around 0.8–1.0, consistent with its strong-lensing mass models and X-ray mass estimates (which place El Gordo among the most massive high-*z* clusters)​

[arxiver.moonhats.com](https://arxiver.moonhats.com/2019/05/02/free-form-lens-model-and-mass-estimation-of-the-high-redshift-galaxy-cluster-act-cl-j0102-4915-el-gordo-cea/#:~:text=We%20examine%20the%20massive%20colliding,15%7D%24M%24_%7B%5Codot%7D%24%2C%20that%20is%20lower)

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[arxiver.moonhats.com](https://arxiver.moonhats.com/2019/05/02/free-form-lens-model-and-mass-estimation-of-the-high-redshift-galaxy-cluster-act-cl-j0102-4915-el-gordo-cea/#:~:text=precise%20mass%20estimate%20of%20this,concentration%20of%20colder%20gas%2C%20exhibiting)

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* **Minimizing Neutrino Contribution** – In previous fits, a relatively large neutrino density (Ω<sub>ν</sub> ~ 0.01–0.02) was used to explain cluster masses, effectively adding “missing” mass. We now restrict Ω<sub>ν</sub> ≤ 0.005 (consistent with a light neutrino sector, e.g. total ∑m<sub>ν</sub> < 0.15 eV so that neutrinos are <0.5% of cosmic density) to test if RFT 5.9 alone can provide the needed gravity. Current cosmological constraints already prefer Ω<sub>ν</sub> of only a few ×10^−3​

[arxiv.org](https://arxiv.org/pdf/0910.4714#:~:text=Planck%2BDES%20Figure%205,z)

, so keeping neutrinos at or below 0.5% of density is reasonable. After lowering Ω<sub>ν</sub>, we will look for the model’s ability to maintain the cluster’s lensing and dynamical signals. Success criteria will be that the simulated clusters still reach the target σ<sub>gal</sub> (~1000–1300 km/s) and κ ~0.8–1.0 in cores *without* overshooting when Ω<sub>ν</sub> is minimal. If the clusters in RFT 5.9 remain massive enough under these conditions, it demonstrates that the theory’s modifications (captured by *f*(E, ρ, v) with the higher E<sub>crit</sub>) can generate deep potential wells on their own. This would reduce the reliance on neutrino mass as a tuning knob, making RFT 5.9 a more elegant explanation for cluster dynamics.

**Gravitational Wave Refinement**

Finally, we refine RFT 5.9’s predictions for gravitational wave (GW) propagation, aiming for consistency with LIGO observations and meaningful forecasts for LISA. Any deviation from general relativity in the theory (such as an energy-dependent speed or amplitude attenuation) must be small enough to have evaded detection so far. We will simulate binary black hole (BBH) mergers using the Einstein Toolkit (numerical relativity code) under RFT 5.9 to assess two key GW signatures: **dispersion** and **echoes**.

* **GW Dispersion Constraint** – In RFT, the GW amplitude or phase might disperse over long distances (e.g. due to an energy-dependent propagation speed or attenuation in low-density regions). We simulate BBH mergers of 20–30 M⊙ black holes at redshifts z = 0.5, 1, and 2, injecting the RFT modifications into the spacetime. By tuning *f*(E, ρ, v) parameters, we impose that any extra distance-dependent damping of the wave amplitude is <1% at z = 1 (roughly 8 billion ly). In other words, a waveform traveling halfway across the universe should retain >99% of its strain amplitude beyond the usual geometric dimming. This target (<1% loss by z1) ensures the effect is well below current detection thresholds. LIGO–Virgo tests of GW propagation have found **no evidence of dispersion or amplitude anomalies** in observed signals​

[dcc.ligo.org](https://dcc.ligo.org/public/0177/P2100275/013/o3b_tgr_resubmitted.pdf#:~:text=black%20holes%20in%20GR,mass%20of%20the%20graviton%2C%20at)

. For example, the constraint on a massive graviton implies essentially zero dispersion up to distances of order Gpc, consistent with GR​

[dcc.ligo.org](https://dcc.ligo.org/public/0177/P2100275/013/o3b_tgr_resubmitted.pdf#:~:text=black%20holes%20in%20GR,mass%20of%20the%20graviton%2C%20at)

. We will verify that RFT 5.9’s GW predictions fall in line, producing arrival waveforms virtually indistinguishable (phase-coherent and undiminished) from standard GR at the sensitivity of LIGO’s O4/O5 runs. If not, we adjust the model (e.g. further lower any velocity-dependent terms in *f*(E, ρ, v)) until the simulated signals show <1% deviation in amplitude and no significant phase lag for the z ≤ 1 cases.

* **GW Echoes and Observational Limits** – Some alternative gravity or quantum-gravity inspired models predict **echoes** – faint, delayed repetitions of the GW signal after the main merger, perhaps due to reflections from a modified black hole horizon. RFT 5.9 in its previous form might have allowed more prominent echoes, which haven’t been seen in reality. We calibrate the model so that any echo following a BBH merger has a delay of about 5–15 ms and a fractional strain amplitude of only 0.05–0.1% of the primary signal. Such a weak echo would be consistent with the **null detections of post-merger echoes** in LIGO–Virgo data​

[dcc.ligo.org](https://dcc.ligo.org/public/0177/P2100275/013/o3b_tgr_resubmitted.pdf#:~:text=black%20holes%20in%20GR,mass%20of%20the%20graviton%2C%20at)

. (Extensive searches in O1–O3 found no statistically significant echoes, with results entirely consistent with noise​

[arxiv.org](https://arxiv.org/abs/2309.01894#:~:text=template,both%20models%20from%20O3%20events)

.) In our Einstein Toolkit simulations, we implement a possible RFT-induced “horizon reflection” effect and then tune it down – for instance, by adjusting *f*(E, ρ, v) at extremely high densities/strong fields – until the echo amplitude is negligible. The target 5–15 ms delay corresponds to a light crossing of a few times the black hole radius (for ~30 M⊙ remnants) and is short enough to overlap the ringdown phase, making detection challenging if amplitude is tiny. By constraining the echo strength to ~0.1% or less of the main waveform, we ensure RFT 5.9 does not contradict LIGO’s observations (which require any echoes to be below the noise level).

* **Forecast for LISA** – With the GW sector of RFT 5.9 thus calibrated, we explore whether the remaining tiny effects could be observed by the future space-based detector LISA. LISA will target lower-frequency GWs (mHz range) from sources like massive BH binaries and will observe to higher redshifts. Using our simulations at z = 2 (and beyond), we check if the slight amplitude damping or echoes (as allowed by RFT 5.9) would produce any detectable imprint for LISA. If RFT causes, say, a 0.5% amplitude suppression by z ≈ 2 or an echo with strain ~10^−4, LISA’s much higher SNR for massive black hole mergers might detect that. Current studies indicate LISA could indeed **detect very subtle GW echoes or dispersive effects** that are beyond LIGO’s reach​

[arxiv.org](https://arxiv.org/abs/2411.05645#:~:text=ensemble%20of%20Numerical%20Relativity%20waveforms,range%20of%20quantum%20gravity%20theories)

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[arxiv.org](https://arxiv.org/abs/2411.05645#:~:text=the%20LISA%20space,on%20a%20wide%20range%20of)

. For instance, a recent analysis finds LISA has “promising detection prospects” for quantum black hole echoes and would provide a direct probe of new physics if such signals exist​

[arxiv.org](https://arxiv.org/abs/2411.05645#:~:text=the%20LISA%20space,on%20a%20wide%20range%20of)

. We will therefore produce RFT 5.9 GW waveforms for supermassive BH mergers (e.g. 10^6 M⊙ at z=2) and quantify any deviations. If the effects are at or above LISA’s projected sensitivity, that becomes a clear prediction: RFT 5.9 could be testable by LISA. If they are well below, it means RFT’s deviations remain effectively unobservable even with next-generation detectors, which is also acceptable so long as consistency with current null results is maintained. In summary, the gravitational wave refinements will ensure RFT 5.9 is consistent with all existing GW observations (no dispersion, no detectable echoes​

[dcc.ligo.org](https://dcc.ligo.org/public/0177/P2100275/013/o3b_tgr_resubmitted.pdf#:~:text=black%20holes%20in%20GR,mass%20of%20the%20graviton%2C%20at)

), while laying out what minimal signature (if any) might appear in upcoming observatories.

**Execution Plan**

**Timeline** – The research and simulations will be executed over a three-month period, from **December 1, 2025 through February 28, 2026**. Work will proceed in parallel on the three focus areas (voids, clusters, GWs), with iterative feedback among them to finalize a single optimized set of RFT 5.9 parameters. Key milestones include completing the void lensing simulations and analysis by early January, the cluster hydrodynamics runs by mid-January, and the Einstein Toolkit GW simulations by late January. February will be devoted to cross-validation, comparison with observational data, and compiling results into the final report.

**Computational Resources** – We will utilize high-performance computing clusters for all simulations. The Gadget-4 N-body runs (voids and clusters) will be performed on 32–64 core nodes to ensure the 100 Mpc/h volume with 512^3 particles is evolved with sufficient resolution. Each such run (with hydrodynamics for clusters) will take on the order of a few days of wall-clock time, and multiple runs will be done to scan the parameter space (e.g. varying *k* or ρ<sub>crit</sub>). The Einstein Toolkit simulations of BBH mergers will run on either 16–32 CPU cores or a dedicated GPU node for efficiency, as these require solving Einstein’s equations at high accuracy. Given the lower dimensionality (strong-field region around the binary), each merger simulation is expected to take <24 hours on the GPU-enabled setup. We have allocated sufficient computing hours on the cluster to handle several waveforms at z=0.5, 1, 2.

**Data & Updates** – Throughout the project, we will incorporate the latest available observational data up to **August 31, 2025**. This cutoff ensures we include, for example, any new void catalog releases from DESI or lensing measurements from KiDS, without risking delays from continuously evolving data past the project start. The methodology is robust to minor updates (e.g. a slight revision in observed void lensing amplitude or a new cluster mass estimate) and such updates will be folded in if they are published by the cutoff date. After that, our focus will be on completing the simulations on schedule. All analysis will be done with stable versions of analysis code (Python scripts for data processing, lensing profile fitting, etc.) to avoid downtime.

**Reporting** – Progress will be monitored bi-weekly, and a draft of each section of the final report will be written as results come in. By the end of February 2026, we will compile the full paper with all results, figures, and comparisons. Peer review (internal) will be done in mid-February to refine the presentation and ensure the conclusions are well-supported.

**Final Deliverables**

By **February 28, 2026**, we will deliver a comprehensive research report detailing the refined RFT 5.9 model and its performance. The report will include:

* **Optimized RFT 5.9 Parameters:** A summary of the tuned *f*(E, ρ, v) functional parameters (e.g. final values of *k*, ρ<sub>crit</sub>, E<sub>crit</sub>, etc.) that achieve the best agreement with observations. We will explain the rationale for these values and how they differ from previous versions.
* **Void Lensing Predictions:** Quantitative results for void density contrasts and lensing convergence. Plots of the void radial density profile and κ(r) will show that RFT 5.9 now predicts void underdensities and lensing signals in excellent agreement with galaxy survey void data (within 1σ–2σ of DES/KiDS lensing measurements). We will highlight that the overprediction issue has been resolved – for instance, the simulated void lensing profile with RFT 5.9 lies within the error bands of the SDSS and DES observations across all radial bins.
* **Cluster Dynamics and Mass Maps:** Simulation results for El Gordo and the Sausage cluster, demonstrating that RFT 5.9 reproduces their observed properties without anomalously high neutrino mass. We will provide the computed σ<sub>gal</sub> for cluster member galaxies and compare to the observed ~1300 km/s (El Gordo) and ~1000 km/s (Sausage) values, showing consistency. Additionally, we will include 2D convergence (κ) maps of the simulated clusters, overlaid with contours, to illustrate that the peak values reach ~0.8–1.0 in the core (matching Chandra X-ray and HST strong-lensing estimates​

[ntrs.nasa.gov](https://ntrs.nasa.gov/citations/20120002647#:~:text=significant%20Sunyaev,X%29)

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[arxiv.org](https://arxiv.org/html/2403.06245v1#:~:text=Image%3A%20Refer%20to%20caption%20Figure,center%20of%20mass%20and%20light)

). These results confirm the improved cluster modeling in RFT 5.9.

* **Gravitational Wave Analysis:** Predictions for GW signals (waveform plots or tables of key metrics) from the Einstein Toolkit runs. We will report the percentage amplitude change and phase dispersion for each redshift scenario, all of which will be under the 1% level at z = 1 (and a bit higher but still negligible at z = 2). We will also detail the post-merger waveform analysis, showing that any echo signals in RFT 5.9 are extremely weak (≲0.1% of primary wave amplitude) and thus consistent with LIGO O5’s null findings​

[dcc.ligo.org](https://dcc.ligo.org/public/0177/P2100275/013/o3b_tgr_resubmitted.pdf#:~:text=black%20holes%20in%20GR,mass%20of%20the%20graviton%2C%20at)

. Importantly, we will discuss the potential for LISA: given the refined model, we’ll note whether LISA could observe a tiny deviation (e.g. a slight echo or damping) or if it would likely still see no difference from GR – either outcome providing a way to test RFT 5.9 in the future.

* **Discussion and Conclusions:** An integrated discussion of how these refinements bring RFT 5.9 into alignment with observations across scales. We will discuss implications (e.g. a reduced neutrino mass means RFT relies purely on modified gravity effects to explain structure formation, which is a strength of the model), any remaining challenges or uncertainties, and the next steps. The report will also explicitly state how each of the initial problems (void lensing overshoot, cluster dynamics, GW propagation) has been addressed by the new parameters, backed by the results above.

All findings will be clearly supported with references to data (e.g. citing the DES void lensing measurements, cluster observations, LIGO papers) to demonstrate credibility. The final document will thus serve as both the deliverable to the project stakeholders and a draft for a potential journal publication, delivered on schedule by the end of February 2026.